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ISOPERIMETRY WITHOUT CURVES OR CALCULUS.

By Professor P. H. PHILBRICK, M. Sc., C. E., Lake Charles, Louisiana.

[Continued from the October Number.]

PROPOSITION III. *Of all triangles formed with two given sides, that in which these sides are perpendicular to each other is a maximum.*

Let ABC and $A'B'C$ be two triangles having $AB = A'B$, BC common, and ABC a right-angle.

Now $A'H < A'B = AB$. Hence the bases being the same, and the altitude of the one less than the altitude of the other, the area is also less.

PROPOSITION IV. *Any rectangle is greater than any rhomboid, if their bases and perimeters are equal.*

Completing the parallelograms $ABCD$ and $A'B'C'D'$, we have, $ABCD > A'B'C'D'$, since they are respectively double the triangles ABC and $A'BC$.

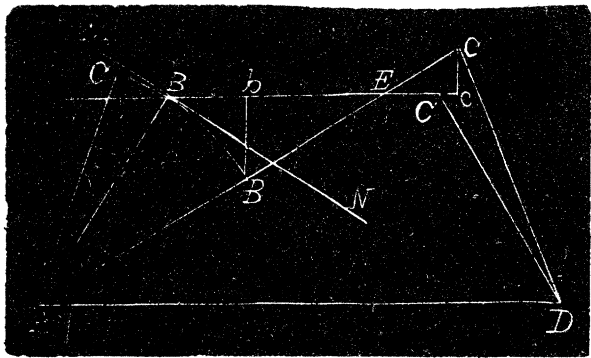
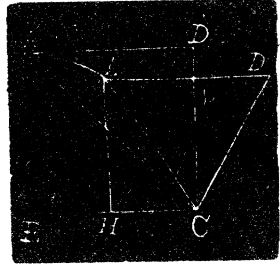
COROLLARY. It follows that any square is greater than any rhombus of equal perimeter.

PROPOSITION V. *Of all quadrilaterals having the same sides, three being equal; that which has equal angles between equal sides, has the maximum area.*

Let $ABCD$ be such a quadrilateral in which $AB = BC = CD$, and $ABC = BCD$. And let us suppose those sides to take the position AB' , BC' and $C'D$, the side AD remaining fixed.

Draw MBN perpendicular to AB . Drop the perpendiculars $B'b$ and $C'c$ on BC and BC' prolonged, and prolong $C'B$ to the left. Make the triangle $ABC' =$ the triangle DCC' . Now angle $B'Bb > NBb$ and angle $C'Ce = C'BA < MBb = NBb$. $\therefore C'Ce < B'E$ and $cE < C'E$; adding gives $bc < B'C'$ or $bC' + Cc < B'C' = BC$.

But $Bb + bc = BC$. $\therefore Bb + bC' > bC' + Cc$ or $Bb > Cc$. And (since $C'Ce < B'Bb$), still more is $C'c < Bb$, and $CC' < BB'$. Hence, triangle $DCC' <$ triangle ABB' . Moreover, since $cC' < bB'$; $C'E < B'E$, $cE < bE$, and still more is $C'E < BE$.



Hence triangle $EC'C' < \text{triangle } EBB'$. But, $ABCD = AB'EC'D + ABB' + EBB'$ and $AB'C'D = AB'ECD + DCC' + EC'C'$.

Hence $ABCD$ is greater than $AB'C'D$.

COROLLARY. It follows, that the quadrilateral of three equal sides, and maximum area, is a trapezoid; that the angles including the fourth side are also equal; that the opposite angles are supplementary; and that the trapezoid is inscriptible.

[To be continued.]

PROFESSOR SYLVESTER'S RECIPROCANTS.

By F. P. MATZ, M. Sc., Ph. D., New Windsor, Maryland.

To those functions of the successive derivatives of y with respect to x , which preserve their form unaltered, except for $dy \propto dx$ as a factor, when the independent and dependent variables x and y are interchanged, Professor Sylvester gave the name of *Reciprocants*.

According to the general theory with respect to the inversion of the independent and dependent variable, we must have the relations:

$$\begin{aligned}\frac{dy}{dx} &= 1 \Big/ \frac{dx}{dy}, \frac{d^2y}{dx^2} = \frac{d}{dx} \left(1 \Big/ \frac{dx}{dy} \right) = \frac{d}{dy} \left(1 \Big/ \frac{dx}{dy} \right) \frac{dy}{dx} = - \frac{d^2x}{dy^2} \Big/ \left(\frac{dx}{dy} \right)^3; \\ \frac{d^3y}{dx^3} &= \frac{d}{dy} \left[- \frac{d^2x}{dy^2} \Big/ \left(\frac{dx}{dy} \right)^3 \right] \frac{dy}{dx} = - \left[\frac{dx}{dy} \cdot \frac{d^3x}{dy^3} - 3 \left(\frac{d^2x}{dy^2} \right)^2 \right] \Big/ \left(\frac{dx}{dy} \right)^5; \\ \frac{d^4y}{dx^4} &= - \left[\left(\frac{dx}{dy} \right)^2 \left(\frac{d^4x}{dy^4} \right) - 10 \frac{dx}{dy} \cdot \frac{d^2x}{dy^2} \cdot \frac{d^3x}{dy^3} + 15 \left(\frac{d^2x}{dy^2} \right)^3 \right] \Big/ \left(\frac{dx}{dy} \right)^7; \text{ etc.}\end{aligned}$$

After these relations are substituted for the various differential coefficients of y with respect to x , in any function of these differential coefficients or derivatives, we are said to have interchanged the independent and dependent variable.

Assume $dy \propto dx = T$, $d^2y \propto dx^2 = A$ | 2, $d^3y \propto dx^3 = B$ | 3, $d^4y \propto dx^4 = C$ | 4, etc.; then, after eliminating the constants in the general equation of the straight line, by the method of differentiation, we obtain $d^2y \propto dx^2 = A$, $= d^2x \propto dy^2 = 0$ (1).

The left-hand member of (1) is Professor Sylvester's first *pure* reciprocant, since it does not involve $dy \propto dx$; and this reciprocant is briefly and typically expressed by A . The third member of (1) represents the reciprocant when the independent and dependent variables x and y are interchanged.